The equations presented in the article *Circular Gravitational Fields* are first order approximate transformations of quantities within circular gravitational fields. Exact transformations can be derived, but first it will be necessary to examine the effect of gravity on energy. Consider a simple experiment where a mass m is moved from frame A to frame B within a gravitational field. The work received from this change in position is W_{AB}. In frame B, some of the material of mass m could be converted into heat energy E (using nuclear fission) and held inside of mass m with insulation. Then, as a separate activity, energy E is extracted from mass m and combined with W_{AB}. The total energy received from the experiment would be $W_{AB} + E$.

Now the experiment is repeated, but this time the identical material conversion to heat energy E is made in frame A. This energy is again kept inside of mass m. Mass m is now moved from frame A to frame B. In frame B, the energy E is extracted from mass m and combined with the energy obtained by the movement from frame A to frame B. This energy received by the movement from frame A to frame B must also be W_{AB}, giving a total energy of $W_{AB} + E$ once again. Any other result would violate the Law of Conservation of Energy. The two experiments have beginning and end states that are the same, so the total energy received from the two experiments must be the same. Therefore the force of gravity on energy E must be the same as the force of gravity on the material mass that was converted into energy E.

The effect of gravity on energy will be used in the following experiment. Figure 8 shows a small

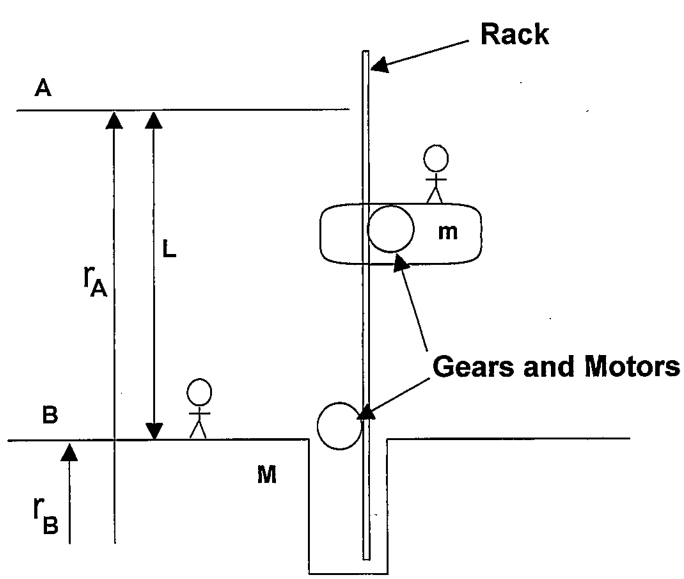

Figure 8

mass m as it is traveling between frame A and B in a gravitational field. The trip is made using a gear and rack, with a motor/generator. Each motor/generator withdraws or adds energy to a battery within it. There are two options for this experiment. One option is to keep the motor located on mass m stopped and to raise and lower m using the motor attached to frame B. Frame B is also mass M producing the gravitational field. The second option is to stop the motor on frame B and raise and lower mass m using the motor that is part of that mass.

First, the option where the mass is raised and lowered by the motor attached to frame B will be examined. This is the "pole work" calculated in the articles *Special Relativity and Gravity* and *Circular Gravitational Fields.*

CYLINDRICAL GRAVITATIONAL FIELDS - POLE WORK

The pole work for a cylindrical gravitational field (also equivalent to a DEOGF) will now be calculated in a new way. Refer to the cylindrically shaped field surrounding the pole mass that generates the DEOGF in the x-Q plane shown in the article *Special Relativity and Gravity*, Figure 1 and Figure 2. This cylindrical gravitational field would exist in a plane perpendicular to the x-axis of the pole. Figure 9 shows the experiment in more detail.

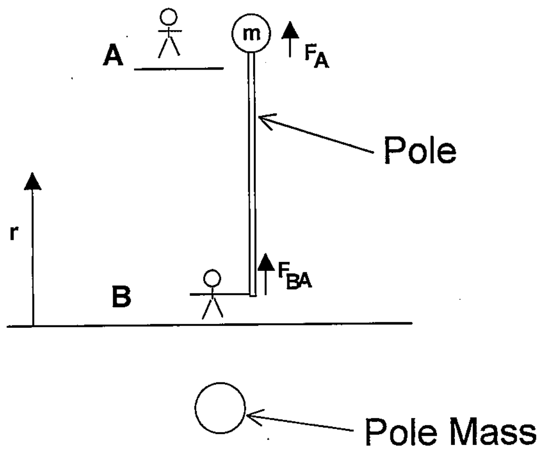

Figure 9

The pole work is calculated assuming the mass starts in frame B. According to (4) from *Special Relativity and Gravity*, the weight of mass m in the two reference frames shown is:

$$F_A = \frac{2Gm\delta}{r_A} \qquad F_B = \frac{2Gm\delta}{r_B} \tag{27}$$

An estimate of the pole work on mass m at some position r has been provided by (12).

$$W_{AB} = F_B(r - r_B) = \left(\frac{2G\delta m}{r_B}\right)(r - r_B) = \frac{mc^2}{r_B}(r - r_B) \qquad (28)$$

From (18) with $E = mc^2$, the force applied by observer B to the bottom of the pole is F_{BA}. The observer lifts m very slowly. Using (28), the exact calculation of the pole work W_{AB} is:

$$W_{AB} = \int_{r_B}^{r_A}\left(\frac{2Gm\delta}{r}\right)\left(1 + \left(\frac{r - r_B}{r_B}\right)\right)dr$$

$$W_{AB} = \int_{r_B}^{r_A}\left(\frac{2Gm\delta}{r_B}\right)dr = \frac{2Gm\delta}{r_B}(r_A - r_B) = F_B L \qquad (29)$$

Because of the form of acceleration defined by (4), the derivation (29) reduces to $W_{AB} = F_B L$, which is also the deduction (12) from the article *Special Relativity and Gravity*. The pole work calculates as expected using this new procedure, so nothing appears to be out of place. But the form of the gravitational field described by (4) and (29) is a special case, as will become evident when the same procedure is used for spherical gravitational fields.

SPHERICAL GRAVITATIONAL FIELDS - POLE WORK

Assume that the pole mass of Figure 9 is now a spherical planet of mass M. The weight of mass m in the two reference frames shown is:

$$F_A = \frac{GmM}{r_A^2} \qquad\qquad F_B = \frac{GmM}{r_B^2} \qquad (30)$$

Now an expression for F_{BA} must be deduced. The assumption for the spherical gravitational field is that it follows the same form as the other gravitational fields and dynamic accelerations.

$$F_{BA} = F_A\left(1 + \frac{W_{AB}}{mc^2}\right) \qquad (31)$$

Using this definition and the approximate W_{AB} (19), the calculation similar to (29) is repeated.

$$W_{AB} = \int_0^L F_{BA}dz = \int_{r_B}^{r_A} F_r\left(1 + \left(\frac{GM}{c^2}\right)\left(\frac{1}{r_B} - \frac{1}{r}\right)\right)dr$$

$$W_{AB} = \int_{r_B}^{r_A}\left(\frac{GmM}{r^2}\right)\left(1 + \left(\frac{GM}{c^2}\right)\left(\frac{1}{r_B} - \frac{1}{r}\right)\right)dr$$

$$W_{AB} = GmM\left[-\frac{1}{r} + \frac{GM}{c^2}\left(-\frac{1}{rr_B} + \frac{1}{2r^2}\right)\right]_{r_B}^{r_A}$$

$$W_{AB} = GmM\left(\frac{1}{r_B} - \frac{1}{r_A}\right) + \frac{G^2M^2m}{2c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)^2 \qquad (32)$$

In (32), the calculation of W_{AB} did not give the expected results. Equation (19) is not exactly correct for a spherical field. When dealing with a cylindrical field or a DEOGF, W_{AB} is a linear function of the displacement within the field. But in the case of a spherical field, the linear function is only an approximation. So, (32) is a better approximation of W_{AB}. Because of this, the process of (32) can be repeated using the new expression of W_{AB}.

$$W_{AB} = \int_0^L F_{BA} dz = \int_{r_B}^{r_A} F_r\left(1 + \left(\frac{GM}{c^2}\right)\left(\frac{1}{r_B} - \frac{1}{r}\right) + \frac{1}{2}\left(\frac{GM}{c^2}\right)^2\left(\frac{1}{r_B} - \frac{1}{r}\right)^2\right) dr$$

$$W_{AB} = GmM\left(\frac{1}{r_B} - \frac{1}{r_A}\right) + \frac{G^2M^2m}{2c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)^2 + \frac{G^3M^3m}{6c^4}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)^3$$

(33)

In (33), W_{AB} is again made more precise than (32). In fact, a pattern is emerging in this calculation. If the calculation process is repeated indefinitely, then:

$$x = \frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$1 + \frac{W_{AB}}{mc^2} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \cdots$$

$$1 + \frac{W_{AB}}{mc^2} = e^x \qquad (34)$$

Equation (34) is the most precise calculation of W_{AB}. It takes into account the nonlinear way that potential energy is accumulated due to a change in position in a gravitational field using Special Relativity. To state this another way, the ratio of forces at the ends of a pole represented by (31) should be corrected to:

$$F_{BA} = F_A \exp(x) \qquad (35)$$

In (31) or (35), the force on the end of the pole F_{BA} is not only supporting the weight of mass m at the top of the pole (F_A), but is also supporting the weight of the potential energy needed to move mass m from frame B to frame A. W_{AB} is that potential energy. But the weight of that potential energy has it own potential energy that must be lifted to frame A. That additional potential energy shows up in (32). And that potential energy addition has its own potential energy that must be lifted to frame A. That additional potential energy is shown in (33), and so on. The weight of mass m at any particular point within the gravitational field is given by (30), but when that mass changes position within the field, the relative force difference is given by (35).

Using (35) the calculation of W_{AB} will be done again.

$$W_{AB} = \int_{r_B}^{r_A} F_r \exp[\left(\frac{GM}{c^2}\right)\left(\frac{1}{r_B} - \frac{1}{r}\right)] dr$$

$$W_{AB} = \int_{r_B}^{r_A} \frac{GmM}{r^2} \exp[\left(\frac{GM}{c^2}\right)\left(\frac{1}{r_B} - \frac{1}{r}\right)]dr$$

$$W_{AB} = mc^2\left[\exp\left\{\frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)\right\} - 1\right] \tag{36}$$

With the new expression for W_{AB} given by (34), the spherical gravitational field now calculates the potential energy using (30), (31) and (35) to give (36). Equations (34) and (36) are now the same, the same situation as occurred in (29). Now, for all three types of gravitational field, the pole work can be calculated directly, including the gravitational attraction on the mass of energy. One assumption that should be recognized is that M is so big, it does not change with the addition or deletion of mass W_{AB}/c^2.

CYLINDRICAL GRAVITATIONAL FIELDS - MASS m WORK

At this point, the second experiment of Figure 8 will be examined, where the energy from the decent of mass m from frame A to frame B is stored within mass m. The motor attached to M is stopped and has no energy input into the experiment. In this case, it will be assumed that the mass of the potential energy W_{AB}/c^2 is significant compared to mass m. The experiment will be done this time by starting mass m at frame A and descending to frame B. As the mass descends very slowly, the potential energy received by the motor (acting as a generator) progressively accumulates within mass m. At any radius r, the force on mass m is:

$$F_r = \frac{2G\delta}{r}\left(m + \frac{W_{AB}}{c^2}\right) \tag{37}$$

From (12), W_{AB} is deduced to be:

$$W_{AB} = F_r(r_A - r) = \left(\frac{2G\delta m}{r}\right)(r_A - r) = \frac{mc^2}{r}(r_A - r) \tag{38}$$

And the new calculation of W_{AB} is:

$$W_{AB} = \int_{r_A}^{r_B} -\left(\frac{2Gm\delta}{r}\right)\left(1 + \left(\frac{1}{r}\right)(r_A - r)\right)dr$$

$$W_{AB} = \int_{r_A}^{r_B} -\left(\frac{2Gm\delta}{r}\right)\left(1 + \left(\frac{r_A - r}{r}\right)\right)dr$$

$$W_{AB} = \int_{r_A}^{r_B} -\left(\frac{2Gm\delta}{r^2}r_A\right)dr = \frac{2Gm\delta}{r_B}(r_A - r_B) = F_B L \tag{39}$$

Once again, the unique properties of this type of gravitational field give the expected result.

SPHERICAL GRAVITATIONAL FIELDS - MASS m WORK

Repeating the second experiment of Figure 8 for spherical gravitational fields, at any position r within the field the weight of mass m is given by:

$$F_r = \frac{GMm}{r^2} \tag{40}$$

As a first approximation, W_{AB} is:

$$W_{AB} = GMm(\frac{1}{r} - \frac{1}{r_A}) \tag{41}$$

However, at position r, this potential energy that is added to m changes the force to:

$$F_r = \frac{GM}{r^2}\left(m + \frac{W_{AB}}{c^2}\right) \tag{42}$$

Therefore, the calculation of W_{AB} becomes:

$$W_{AB} = \int_{r_A}^{r_B} -\left(\frac{GmM}{r^2}\right)\left(1 + \left(\frac{GM}{c^2}\right)\left(\frac{1}{r} - \frac{1}{r_A}\right)\right)dr$$

$$W_{AB} = GmM\left(\frac{1}{r_B} - \frac{1}{r_A}\right) + \frac{G^2M^2m}{2c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)^2 \tag{43}$$

Equation (43) is the same as (32). This new equation for W_{AB} can be substituted into (42) to get a new value of F_r and an equation equal to (33) will result. Following the same logic as applied to get (34), the final expression for W_{AB} for a mass descending from frame A and storing the potential energy gain within itself is (34). For either experiment described for Figure 8, the same expression for W_{AB} results.

Due to the Law of Conservation of Energy, this work W_{AB} that is contained within mass m must be the same as the energy gained when the mass falls from frame A to frame B. Whether the energy obtained is electrical energy stored within a battery or kinetic energy acquired during free fall, the two energies must be equal. That same amount of energy must be required to raise mass m back to reference frame A, whether this task is done by pole work from frame B or whether the battery in mass m is given an electrical charge of W_{AB} and mass m pulls itself back to reference frame A.

This also means that a more exact versions of the relative quantities (21), (22) and (23) from the article *Circular Gravitational Fields* are:

$$\frac{F_B}{F_A} = \exp\left[\frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)\right] \tag{44a}$$

$$\frac{g_B}{g_A} = \exp\left[\frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)\right] \tag{44b}$$

$$\frac{t_A}{t_B} = \exp\left[\frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)\right] \tag{44c}$$

Notice in (44) that length coordinates have no transformation within the gravitational field. All gravitational pole displacements are measured equally in all reference frames regardless of the type of field or direction of displacement. Length contraction is strictly an effect produced by reference frames having a relative velocity.

A further demonstration of this is shown in Figure 10, where there are three observers at three different vertical locations in a gravity field. The observers wish to know how measured vertical distances at each location compare. In other words, as observer A displaces a pole up or down a given distance, do the observers at the other locations see the same displacement? To find out, observer A uses two identical poles as shown. It can be assumed in this experiment that the poles have mass and weight. Observer A moves one pole up slightly and one down the same amount. Since the center of gravity of the two poles together does not move, there is no net energy expended to accomplish the movement of both poles.

In the situation shown in Figure 10, the observers at B and C now apply clamps to the ends of the two poles, so that the positions of the ends are fixed. The observer at A then rotates the poles until they are horizontal in reference frame A. This rotation does not require any energy output by observer A, as the center of gravity of the poles has not moved. In this horizontal position, the observers are assured that L_B and L_C are the same. If they were not the same, the identical poles would now not be identical. They would be of differing lengths with a stress energy in the area between the two clamped ends. This stress energy would be created by the experiment because the observers have been careful to do an experiment that does not require any input of energy.

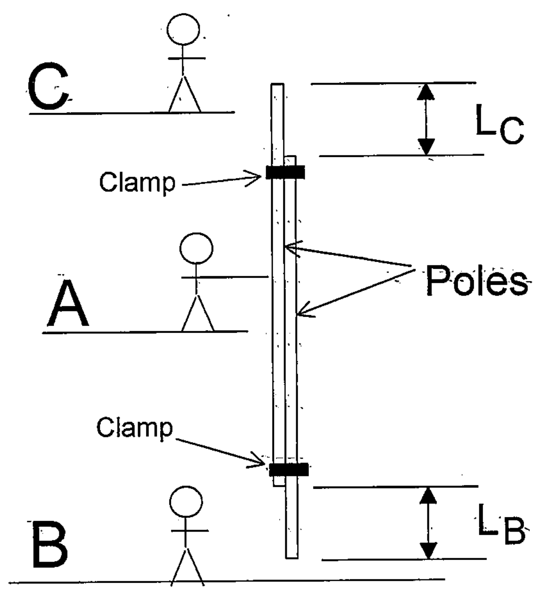

Figure 10

Additional interesting points can be made about gravity fields and energy. In Figure 11, two objects labeled C and D are attached to the ends of the shaft sections as shown. A field attraction exists between C and D. This field could be gravitational, magnetic or electrostatic in nature. A motor/generator on one shaft moves mass C at a constant velocity, as shown. The friction collar applies a force to the other shaft that exactly equals the attraction force between the masses, allowing the shaft to be pulled through the collar at the same velocity as the motorized shaft. As this shaft moves to the right, the distance r between the masses will stay constant. Energy transfers from the motor (and battery) to the friction collar.

Energy is transferred through fields but there is no easily defined mechanism to do this. If C were mechanically fastened to D, the energy transfer is conceptually easier to understand. But Figure 11 shows a transfer of energy across an empty space between C and D. In other words, the experiment of Figure 11 helps to conceptually define whether or not a "field" is a "thing". It doesn't seem to have any of the usual properties of "things" as we think of them in the Newtonian world. It doesn't have any "substance" that we can hold or observe. But energy crosses it in predictable ways, whatever composition it has.

This leads to one more property of these things called fields: fields have mass. In order to display this, again consider Figure 11. This time, the friction collar holds object D in a fixed position. The motor is turned on and energy E is once again expended. This time object C is moved away from object D and distance r is increased. Energy E has disappeared from the battery and has been converted into potential energy of the field.

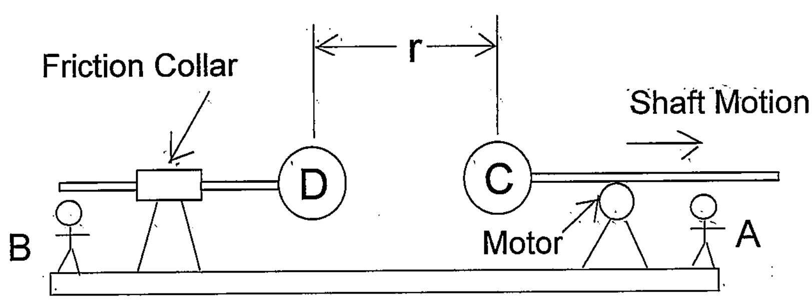

Figure 11

So, consider the entire mechanism of Figure 11 out in space away from other objects before energy E is expended by the motor. Now a vertical force is applied to the bottom of the platform pushing the platform upwards. The entire mechanism will react to acceleration in a known way. Newton's Law, F = ma, can be applied to the whole mechanism, including the mass of energy stored in the motor's battery. The total mass of the system will include all the masses of the individual material parts, as well as the mass E/c^2. The equation of motion of the platform, with energy E stored in the motor's battery, obeys Newton's Laws of motion and the Law of Conservation of Momentum.

Now operate the motor and send energy E into the field between objects C and D by extending distance r. The mass of the battery is now reduced by the loss of this energy. The vertical force is once again applied to the bottom of the platform. In this case, the reaction of the platform with potential energy E stored in the field must be the same as when energy E was in the motor's battery (or heat in the friction collar). The mechanism of Figure 11 is an isolated system and must react to Newton's Laws or the Law of Conservation of Momentum in exactly the same way for any of these situations.

Energy E must have mass equivalent to E/c^2 whether this energy is stored in the battery, friction collar or in the field. Since fields extend in all directions with unlimited dimensions, any object's field must contain potential energy mass which is the compilation of all the potential energies of every object in the universe affected by that field.

Now repeat that last experiment in a gravitational field. As drawn in Figure 11, imagine a device supports the platform from the bottom, with a DEOGF pulling the platform downward. Before energy E is expended by the motor, the platform including E has a weight. Now take energy E from the battery, increase distance r and this energy is stored as potential energy in the field. The weight of the platform mechanism as a whole must be the same. If it is not the same, then the Law of Conservation of Energy can be broken.

To shown how to create energy with this experiment, first notice that the vertical gravitational field will have no effect on the horizontal force between C and D. No matter where the platform is in the vertical field, the force relationship between C and D is the same. So, the weight of the platform pushes the device downward when energy E is stored in the battery. This downward motion produces energy E' which is stored in the device. Now expend energy E from the battery and store it in the field between C and D. By assumption, the platform now weighs less. Take some of energy E' and use it to raise the platform back up to its original height. Then, the potential energy E in the field is allowed to run the motor backwards (acting as a generator) and puts energy E back into the platform battery. The experiment is now back to its original state but the remainder of energy E' stored in the device has been created. Since energy cannot be created, the weight of E when stored as potential energy in the gravity field between C and D must be the same as the weight of any other mass. The potential energy in gravitational fields has mass and weight.

In summary, Relativity Theory recognizes that energy has mass and this makes the analysis of simple gravitational experiments more complicated. The more exact gravity transformations (44) are the result. Since gravitational fields store potential energy, they have mass and weight like any other energy storage device.