

A simple formula for the force between magnetic charges is Gilbert's Model:

$$F = \frac{Uq_1q_2}{R^2} \quad (1)$$

F - Magnetic force between two "charges"

U - A Permeability Constant = $\mu / 4\pi$

μ - permeability

q_1, q_2 - the two charges experiencing the force F

R - the distance between the two charges

The Uniform Linear Magnetic Field

The magnetic field of (1) is approximately spherically shaped around each of the charges. However, it will be useful to examine experiments using a magnetic field which is described by a straight line orthogonal coordinate system. This type of field is created using (1) and specifying that one of the charges is distributed along a rod of infinite length. This configuration is shown in Figure 1.

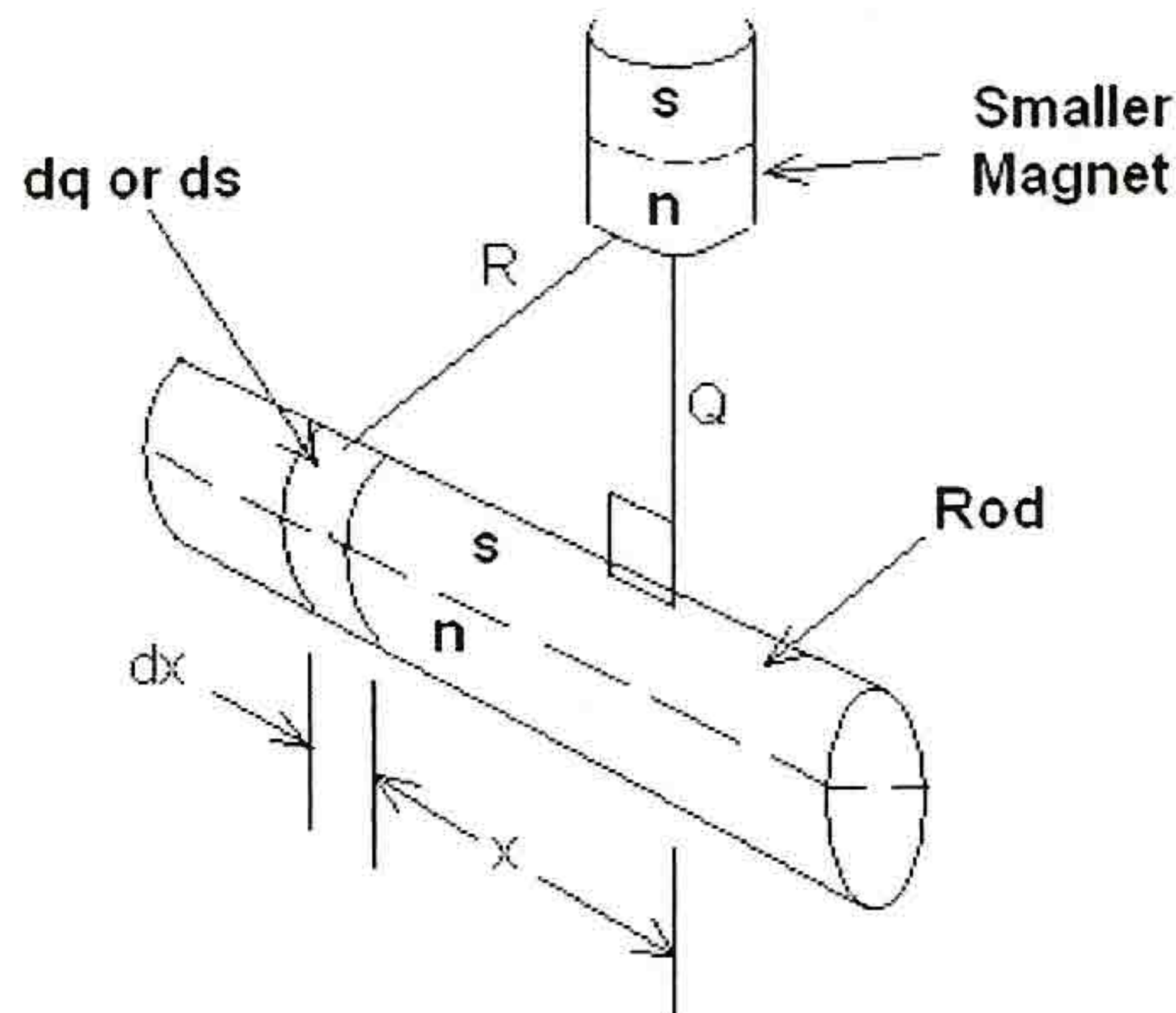


Figure 1. Infinitely long rod magnet creating a uniform linear magnetic field.

The magnets in Figure 1 have the charge distributions (poles) designated by the usual letters "n" and "s", with all the s charges on the top of the rod (near the smaller magnet) and all the n charges lined up along the bottom of the rod. The smaller magnet is arranged so that it is being attracted by the rod magnet, though the analysis could also apply for the opposite arrangement where the two magnets repel each other.

This arrangement can be analyzed using (1) for each of the four pairs of charge interactions (the n and s charges on the smaller magnet each being affected by the n and s

charges on the rod magnet). To begin, just one case of these four will be derived and the other three cases are just variations of the first. The case to be examined is the n charge on the smaller magnet and the s charges on the rod magnet.

The magnetic field associated with this infinite rod is found by first specifying that charge s is an infinitesimal charge ds within the rod. The individual magnetic fields of all the ds charges will then be added up to give the total magnetic field of the rod.

The rod charge distribution is a uniform density of δ and will have a resulting infinitesimal charge $ds = \delta dx$. Infinitesimal charge ds is located a distance x from the n charge of the smaller magnet. Charge n is located a distance Q above the rod. Using (1), the resulting magnetic force dF is:

$$dF = \frac{Un \delta dsx}{Q^2 + x^2} \quad (2)$$

When viewed in the Q-x plane, the force dF can be seen to contribute to the force df (in the Q direction) between charge n and the rod. This is shown in Figure 2.

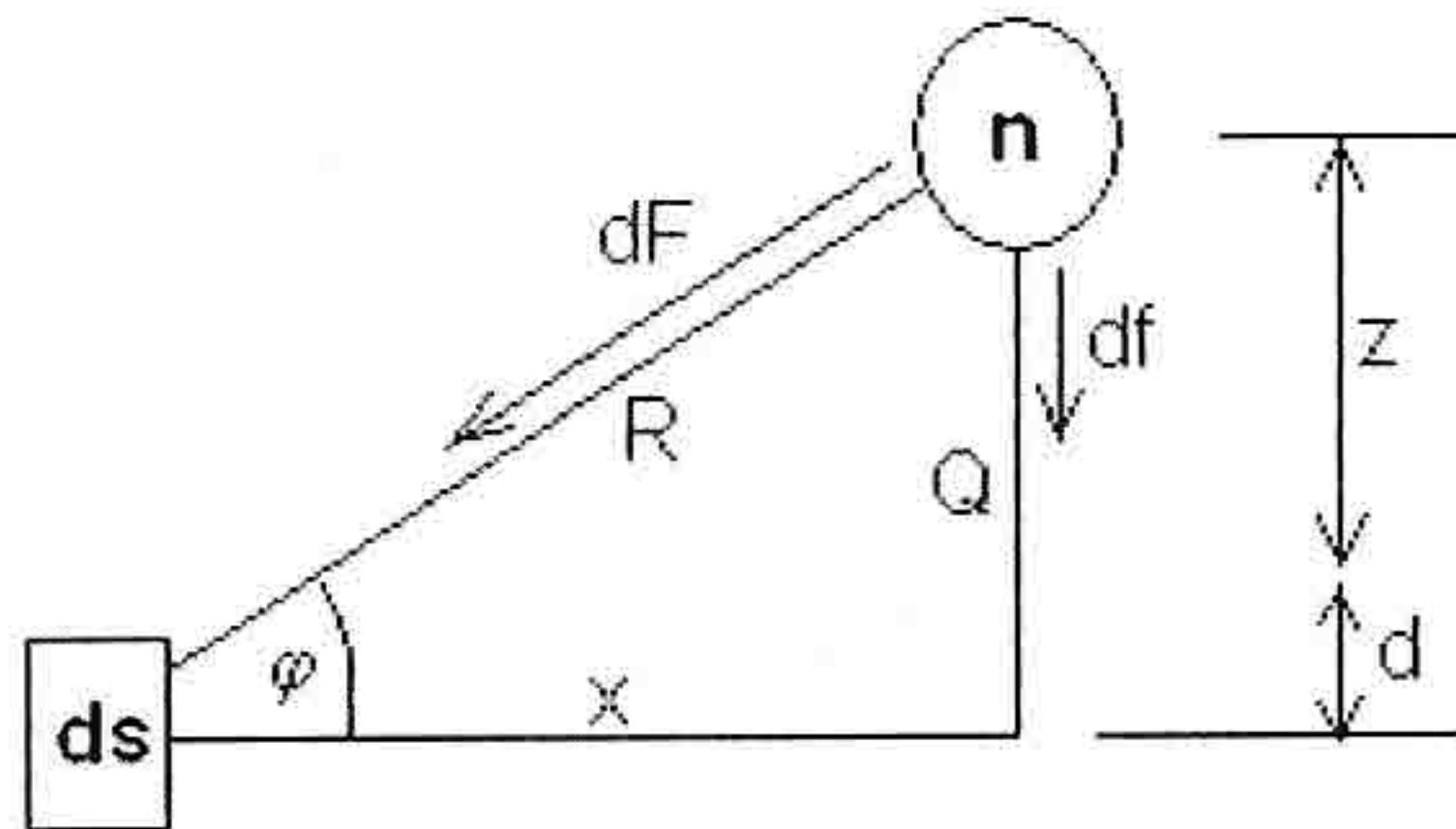


Figure 2. Q-x plane view of the forces.

The resulting equation for the force between charge n and the pole is:

$$df = dF \sin \phi = dF \left(\frac{Q}{\sqrt{Q^2 + x^2}} \right)$$

$$f = 2Un \delta Q \int_0^{\infty} \frac{dx}{(Q^2 + x^2)^{3/2}}$$

$$f = \frac{2Un\delta}{Q} = \frac{2Un\delta}{z+d} \quad (3)$$

The distance Q is divided into two parts, z and d , defined in Figure 2. If a constant j is defined to be the magnetic acceleration divided by c , then the acceleration in the z direction can be defined as:

$$\frac{f}{2Un\delta} = \frac{\frac{1}{d}}{\frac{z}{d} + 1} = \frac{j_z}{c} \quad \text{for any } z$$

$$\frac{j_0}{c} = \frac{1}{d} \quad \text{for the case } z = 0$$

$$j_z = \frac{j_0}{1 + \frac{j_0 z}{c}} \quad (4)$$

The value of j can also be positive or negative, with positive indicating an attractive force and negative indicating a repellant force. Equation (4) is a magnetic acceleration that is in the same form as the dynamic acceleration (21) in the article *The Acceleration Law* and the gravitational acceleration (4) in the article *Special Relativity and Gravity*.

This special magnetic field will be referred to as a Dynamically Equivalent Orthogonal Magnetic Field (DEOMF). It is orthogonal because the magnetic force exists uniformly along the x-axis in the z direction.

Further Definition of the Experimental System

The configuration of Figure 1 can be altered so that experiments can be simplified. The magnetic dipole can be made approximately equal to the simpler gravitational field structure. See Figure 3.

In Figure 3, the infinitely long rod magnet has been replaced by and infinitely long plate magnet. In this configuration, the n and s charges are separated by a greater distance d_3 than they were on the rod in Figure 1. A second plate magnet is offset a distance d_4 from the first plate magnet. Also, its charges are reversed compared to the first plate magnet.

Two smaller magnets are lined up over top of the plate magnets in an arrangement that has both smaller magnets being attracted to both plate magnets. The total charges on the plate magnets are assumed much stronger than the charges on the smaller magnets. Distances d_1 and d_2 are responsible for the magnet force to the plate magnets. If d_1 and d_2 are significantly smaller than d_3 and d_4 , then the experiment approximates a single

smaller magnetic charge being attracted to a single plate (rod) charge distribution. A single magnetic charge calculation can reasonably describe the character of the magnetic field, much as a single calculation describes a gravity field.

The smaller magnets are shown lined up over top of one another, but this is just for clarity in the figure. There can be significant distance (x direction) between them, so that they don't interfere with each other. This may not be necessary if their magnetic charges are weak in comparison to the plate charges, in the same way that objects attracted to the earth are not assumed to gravitationally attract each other in a gravitational experiment.

This configuration will make it easier to understand the configuration of other experiments in this article.

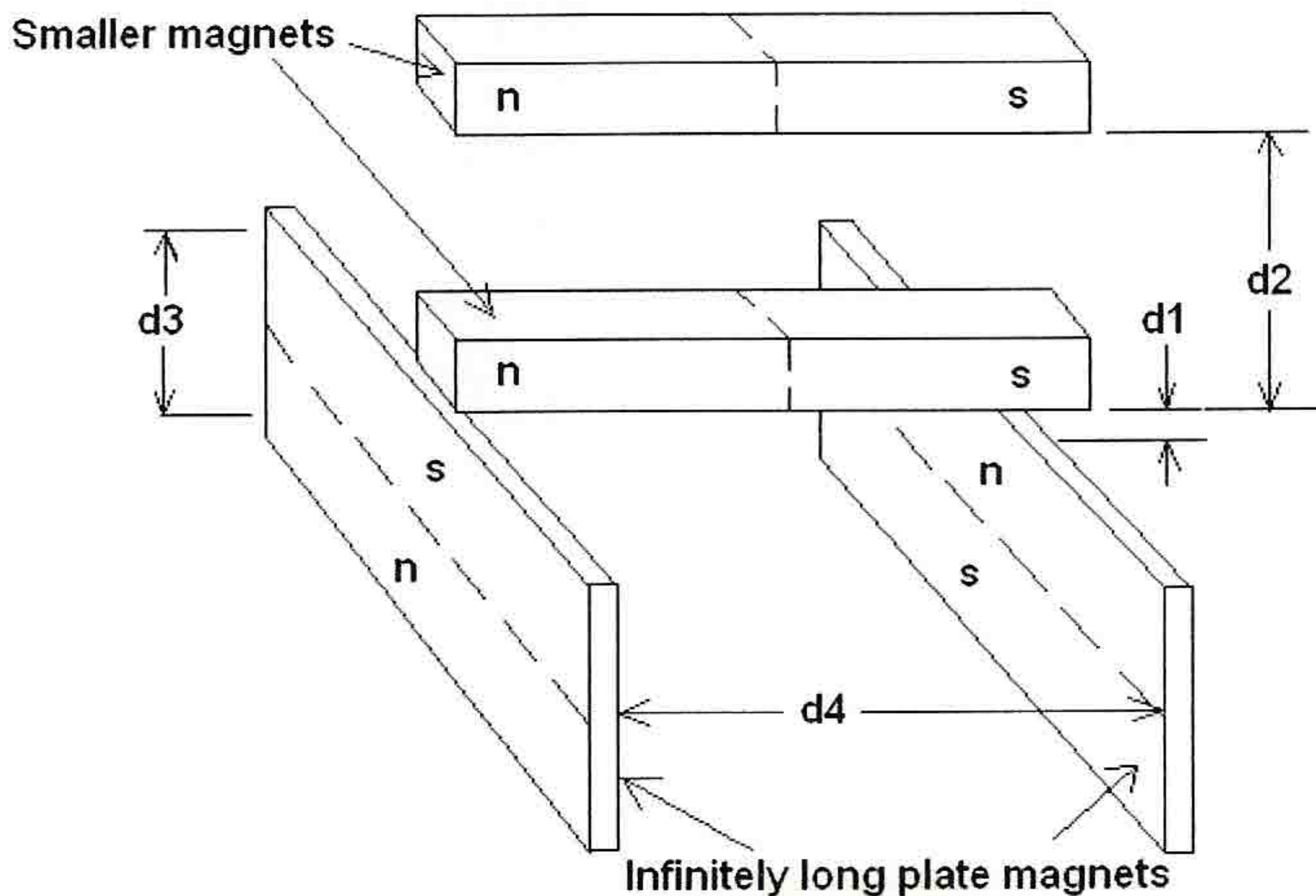


Figure 3. Alternate experimental configuration.

Force Transformation in a Magnetic Field

Consider a thought experiment where an object B has charge n and is stationary in reference frame B at a distance d above the rod magnet. An identical object A is stationary in reference frame A at a distance z above frame B. These two magnetic objects are far enough apart horizontally (in the x direction) so that they have an insignificant effect on each other in the experiment. Due to the different distances to the rod magnet, the magnetic force felt by someone holding the objects would be different. These forces are:

$$F_B = mj_B c \quad \text{for object B} \quad F_A = mj_A c \quad \text{for object A}$$

$$F_A = F_B \left(\frac{j_A}{j_B} \right) = \frac{F_B}{1 + \frac{j_B z}{c}} \quad (5)$$

Now assume that object A falls from frame A to frame B. The kinetic energy generated by this fall is KE. To find KE, note that object A becomes an inertial reference frame as soon as it starts to fall. Frame B accelerates towards object A with constant acceleration j_B . If the time interval that object A sees for the fall is t_A' , then:

$$z = \frac{c}{j_B} \left[\left(1 + (j_B t_A')^2 \right)^{1/2} - 1 \right]$$

$$j_B t_A' = \left[\left(1 + \frac{z j_B}{c} \right)^2 - 1 \right]^{1/2} \quad (6)$$

The velocity that object A has relative to frame B at the moment of impact is β_A and :

$$\beta_A = \frac{j_B t_A'}{\sqrt{1 + (j_B t_A')^2}} \quad (7)$$

The kinetic energy of object A as it impacts frame B is:

$$KE = \frac{mc^2}{\sqrt{1 - \beta_A^2}} - mc^2$$

$$\frac{1}{\sqrt{1 - \beta_A^2}} = 1 + \frac{z j_B}{c}$$

$$KE = mc z j_B \quad (8)$$

Now lets assume object A does not fall from frame A. Instead, it is lowered by observer B with a pole to frame B. Observer B does not know what force he will feel on his end of the pole. He therefore assumes this force will vary with z and calls it F(z). The work gained by observer B during this task will be W_{AB} .

