

A simple and accurate formula for the force of gravity is Newton's Gravitation Law.

$$F = \frac{Gm_1m_2}{R^2} \quad (1)$$

F - Gravitational force between two masses

G - Gravitational Constant

m_1, m_2 - the two masses experiencing the force F

R - the distance between the two masses

THE UNIFORM LINEAR GRAVITATIONAL FIELD

The gravitational field of (1) is approximately spherically shaped around each of the masses. However, it will be useful to examine experiments using a gravitational field which is described by a straight line orthogonal coordinate system. This type of field is created using (1) and specifying that one of the masses is a pole of infinite length. The experiment is shown in Figure 1.

The gravitational field associated with this infinite pole is found by first specifying that mass m_2 is an infinitesimal mass dm within the pole. The individual gravitational fields of all the dm masses will then be added up to give the total gravitational field of the pole.

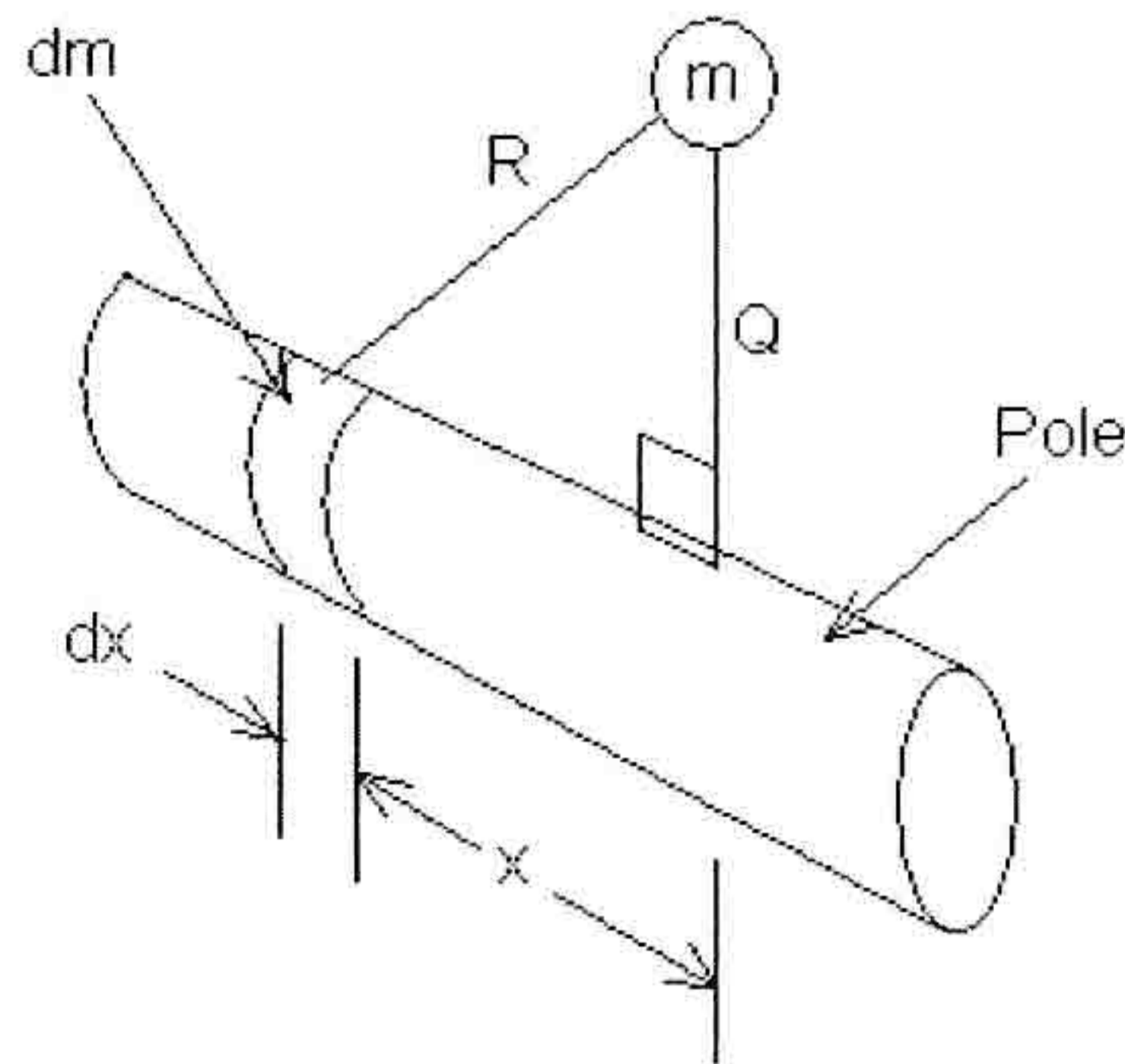


Figure 1. Infinitely long pole mass.

The pole has a density of δ and will have a resulting infinitesimal mass $dm = \delta dx$. Infinitesimal mass dm is located a distance x from mass m . Mass m is located a distance Q above the pole. Using (1), the resulting gravitational force dF is:

$$dF = \frac{Gm \delta dx}{Q^2 + x^2} \quad (2)$$

When viewed in the Q-x plane, the force dF can be seen to contribute to the force df (in the Q direction) between mass m and the pole. This is shown in Figure 2.

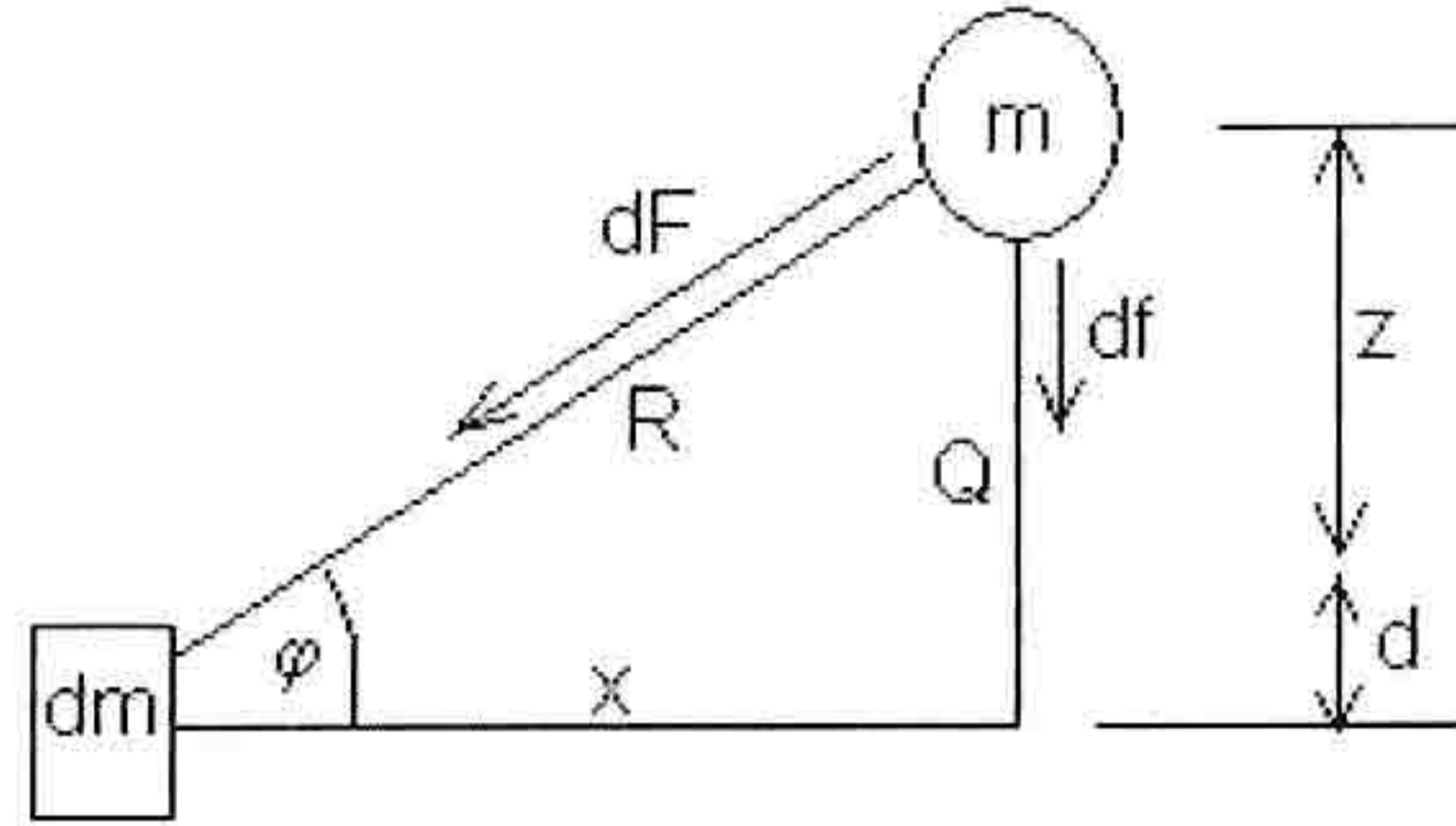


Figure 2. Q-x plane view of the forces.

The resulting equation for the force between mass m and the pole is:

$$df = dF \sin \phi = dF \left(\frac{Q}{\sqrt{Q^2 + x^2}} \right)$$

$$f = 2Gm \delta Q \int_0^{\infty} \frac{dx}{(Q^2 + x^2)^{3/2}}$$

$$f = \frac{2Gm \delta}{Q} = \frac{2Gm \delta}{z + d} \quad (3)$$

If a constant g is defined to be the gravitational acceleration divided by c , a new quantity of acceleration in the z direction can be defined as:

$$\frac{f}{2Gm \delta} = \frac{1}{\frac{z}{d} + 1} = \frac{g_z}{c} \quad \text{for any } z$$

$$\frac{g_0}{c} = \frac{1}{d} \quad \text{for the case } z = 0$$

$$g_z = \frac{g_0}{1 + \frac{g_0 z}{c}} \quad (4)$$

Note that $\delta = 6.733 \times 10^{26}$ kg/m will make $2G\delta/c = 1$. Equation (4) is a gravitational acceleration that is in the same form as the dynamic acceleration (21) in the article *The Acceleration Law*. The distance Q is divided into two parts, z and d, defined in Figure 2. At distance d, the acceleration g_0 is equivalent to α_B From (21). At distance z above d, the acceleration g_z is equivalent to acceleration α_A from (21) with $L = z$.

This special gravitational field will be referred to as a Dynamic Equivalent Orthogonal Gravitational Field (DEOGF). It is orthogonal because the force of gravity exists uniformly along the x-axis in the z direction. All experiments in the x-z plane simulate a dynamic acceleration in the z direction.

FORCE TRANSFORMATION IN A GRAVITATIONAL FIELD

Consider a thought experiment where an object B has mass m and is stationary in reference frame B at a distance d above the pole mass. An identical object A is stationary in reference frame A at a distance z above frame B. Since the gravitational acceleration of objects A and B is different, then the gravitational force felt by someone holding the objects would be different. These forces are:

$$F_B = mg_B c \quad \text{for object B} \quad F_A = mg_A c \quad \text{for object A}$$

$$F_A = F_B \left(\frac{g_A}{g_B} \right) = \frac{F_B}{1 + \frac{g_B z}{c}} \quad (5)$$

Now assume that object A falls from frame A to frame B. The kinetic energy generated by this fall is KE. To find KE, note that object A becomes an inertial reference frame as soon as it starts to fall. Object B accelerates towards object A with constant acceleration g_B . If the time interval that object A sees for the fall is t_A' , then:

$$z = \frac{c}{g_B} \left[\left(1 + (g_B t_A')^2 \right)^{1/2} - 1 \right]$$

$$g_B t_A' = \left[\left(1 + \frac{zg_B}{c} \right)^2 - 1 \right]^{1/2} \quad (6)$$

The velocity that object A has relative to object B at the moment of impact is β_A and :

$$\beta_A = \frac{g_B t_A'}{\sqrt{1 + (g_B t_A')^2}} \quad (7)$$

The kinetic energy of object A as it impacts frame B is:

$$KE = \frac{mc^2}{\sqrt{1 - \beta_A^2}} - mc^2$$

$$\frac{1}{\sqrt{1 - \beta_A^2}} = 1 + \frac{zg_B}{c}$$

$$KE = mczg_B \quad (8)$$

Now lets assume object A does not fall from frame A. Instead, it is lowered by observer B with a pole to frame B. Observer B does not know what force he will feel on his end of the pole. He therefore assumes this force will vary with z and calls it F(z). The work gained by observer B during this task will be W_{AB} .

$$W_{AB} = \int_z^0 F(z) dz \quad (9a)$$

$$W_{AB} = F_{ave} z \quad (9b)$$

The value of F(z) would be applied for an incremental distance dz to give the expression for work in (9a). Another way to calculate the work would be to take the average force during the task F_{ave} and multiply it by z, as is shown in (9b). But, knowing $KE = W_{AB}$ gives:

$$F_{ave} = mcg_B = F_B \quad (10)$$

Equation (10) is true no matter what the value of z is. For any value of z, observer B always feels average force F_B on his end of the pole. Therefore, observer B always feels

constant force F_B on his end of the pole. This force will be called F_{BA} , and object A exerts force F_A on the opposite end of the pole.

$$F_{BA} = F_A \left(\frac{g_B}{g_A} \right) = F_A \left(1 + \frac{g_B z}{c} \right) = F_B \quad (11)$$

THE FORCE OF GRAVITY ON ENERGY

Consider an experiment where a mass is moved from frame A to frame B using a pole. The weight of the mass in frame B is F_B . This movement happens slowly so that no dynamic effects are present. The work received from this experiment is W_{AB} and:

$$W_{AB} = F_B z \quad (12)$$

In frame B, after the mass has arrived, some of the material of the mass is converted into heat energy H (using nuclear fission) and held inside of the mass with insulation. Then, as a separate activity, energy H could be extracted from the mass and combined with W_{AB} . The total energy gained from the experiment is $W_{AB} + H$.

Now the experiment is repeated, but this time the identical material conversion to heat energy H is made in frame A. This energy is again held inside of the mass. The mass is again moved from frame A to frame B. At the frame B location, the energy H is extracted from the mass and added to the energy obtained from the movement of the mass from frame A to frame B. The total energy must again be $W_{AB} + H$ from the Law of Conservation of Energy. The beginning and end states of the experiment are the same, so the total energy received from the experiment must be the same. If W_{AB} is the same for both experiments, then the force of gravity on the energy H must be the same as the force of gravity on the material that was converted into H. Similar experiments could be performed on any other type of energy with the same result.

HORIZONTAL FORCE IN A GRAVITATIONAL FIELD

In Figure 3, observer B pushes on a long board with a hole in it so that it passes around the pole mass. The same board is part of an identical experiment on the opposite side of the pole mass (not shown), which is positioned to eliminate any rotational movement or torque effects in the experiment. Horizontal force F_B is applied by observer B and horizontal force F_A is felt by observer A in frame A. The spring in frame A is compressed, clamped in the compressed position and lowered to frame B by observer B using a pole.

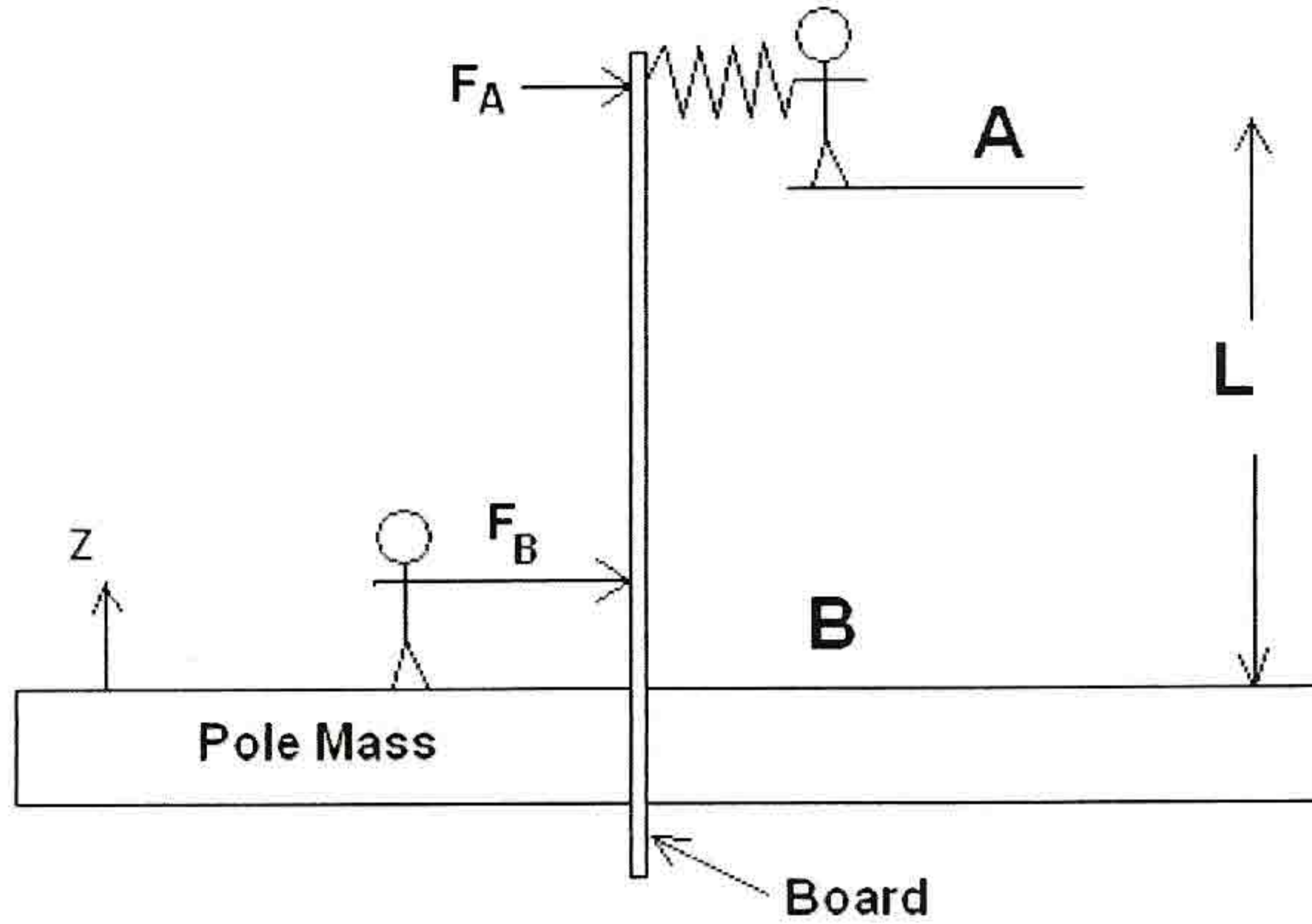


Figure 3. Horizontal force experiment.

The spring has a force-displacement relationship of $F = kdx$, where k is the spring constant and dx is the displacement. When the board compresses the spring, a potential energy E is stored in the spring while in frame A. When the spring has been moved to frame B, the same energy and spring force must be present there. The work gained by lowering the spring down to frame B is:

$$W_{AB} = \left(\frac{E}{c^2} \right) g_B cL \quad (13)$$

The energy initially expended by observer B on the board is $\frac{1}{2} F_B dx$ and the energy stored in the spring is $E = \frac{1}{2} F_A dx$. When the spring is at frame B, the Law of Conservation of Energy gives:

$$\begin{aligned} \frac{1}{2} F_B dx &= E + W_{AB} = \frac{1}{2} F_A dx + W_{AB} \\ \frac{F_B}{F_A} &= 1 + \frac{g_B L}{c} = 1 + \frac{W_{AB}}{E} \end{aligned} \quad (14a)$$

