One implication of the article *Circular Gravitational Fields* is that Special Relativity does not support the concept of 'black holes'. For many years, theories on black holes have been popular in astrophysics. A poorly calculated Schwarzschild Metric is the starting point for these theories. The errors in the Schwarzschild Metric responsible for black holes will be shown, as well as the correct form of the equation.

Any solution to the General Theory of Relativity is a difficult problem. However, the thought experiment examined by the Schwarzschild Solution [1] is not a difficult problem to solve in Special Relativity. The Schwarzschild Metric is just the invariant interval for a single, non-rotating planet in an otherwise empty universe. Schwarzschild used a polar coordinate system for his paper and his original coordinate system is given in Figure 12.

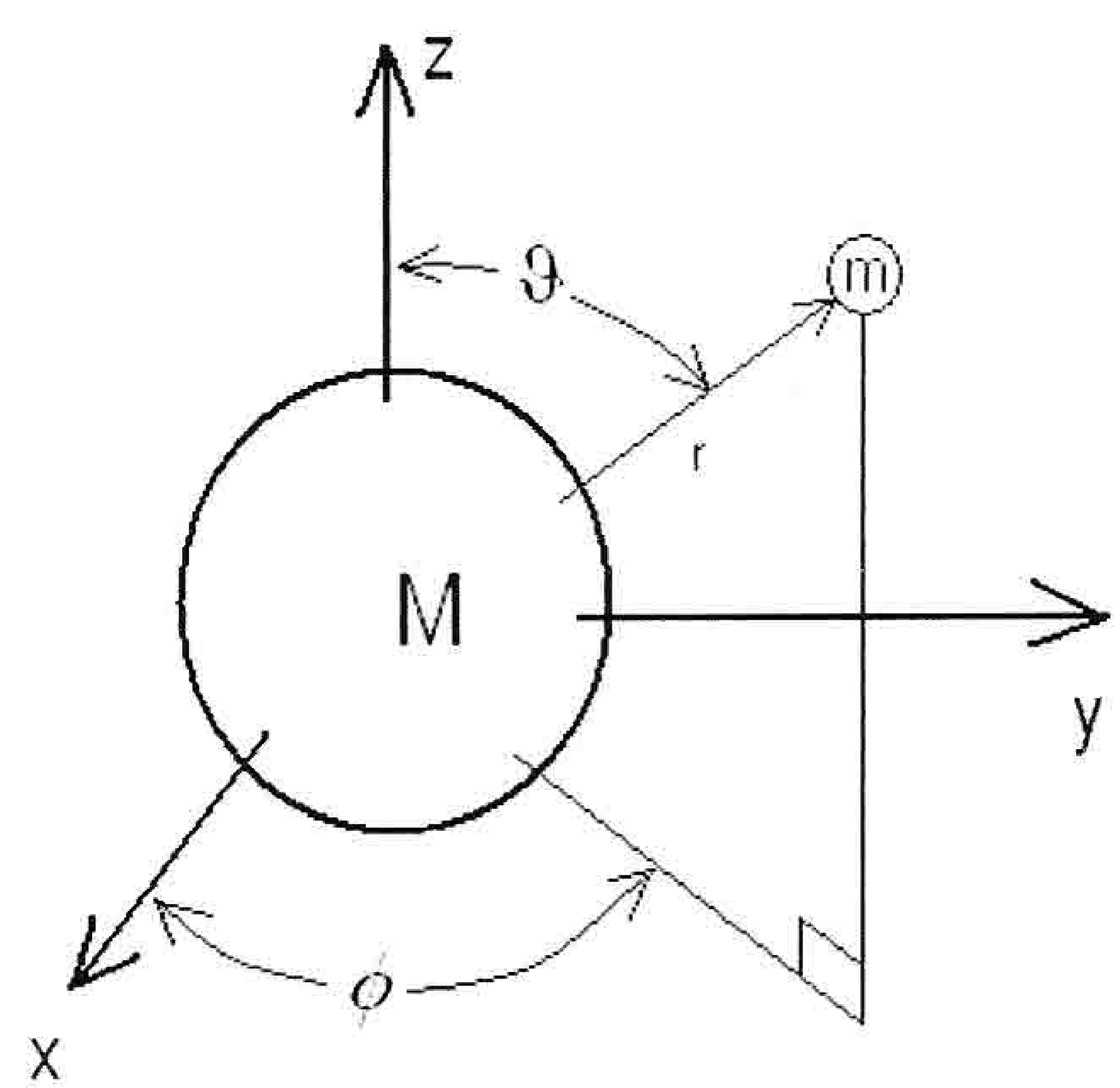

Figure 12

In Figure 12, a large mass M is shown with another mass m at position r. Mass m is so small that it has a negligible effect on the gravitational field of M.

PROBLEM STATEMENT

It is desired to find the invariant interval of the space surrounding M. The invariant interval ds is formed from four position coordinates (including time as a coordinate) describing the location of events in either the linear or polar coordinate systems.

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{45a}$$

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\vartheta^2 - r^2 \left(\sin^2 \vartheta\right) d\phi^2 \tag{45b}$$

In (45), c is the speed of light. For reference, the Schwarzschild Metric commonly cited in current tests, such as Antoci [2], is:

$$ds^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\vartheta^2 - r^2 \left(\sin^2 \vartheta\right) d\phi^2 \tag{46}$$

To go from (45b) to (46), a massless framework is constructed around mass M. This framework has radial and circumferential members simulating the polar coordinate system of Figure 12. Observers from M carry meter sticks out onto this framework and carefully mark off identical intervals throughout the entire structure. Identical clocks are also placed at regular intervals along the structure.

At a location approximating $r = \infty$, mass m is located. Mass m has its own framework, marked off with clocks and meter sticks identical to the ones used for mass M. The form of the m framework is that of a purely inertial system, but this framework only extends a short distance surrounding m. The M and m observers will note that the intervals on each framework are identical when m is at this far away location. They also note that the clocks on each framework proceed at the same rate.

$$dt_M = dt_m$$

$$dr_M = dr_m$$

$$ds^2 = c^2 dt_M{}^2 - dr_M{}^2 - r_M{}^2 d\vartheta_M{}^2 - r_M{}^2 \left(\sin^2 \vartheta_M\right) d\phi_M{}^2 \tag{47a}$$

$$ds^2 = c^2 dt_m{}^2 - dr_m{}^2 - r_m{}^2 d\vartheta_m{}^2 - r_m{}^2 \left(\sin^2 \vartheta_m\right) d\phi_m{}^2 \tag{47b}$$

In a universe with only one significant mass M, the gravitational field extends forever in all directions. There is no actual point $r = \infty$, but only a very large value of r where (47) is reasonably accurate. Now, a rather long period of time passes and mass m eventually falls toward M due to the gravitational attraction. At a point where m has a significant velocity v, the observers on each framework can once again make their measurements. The m observers have noted that they are still an inertial reference frame and conclude nothing has occurred to modify the m framework. However, the m observers note that the M intervals that they measure are now shorter and the M clocks appear to be running slow. These results can be incorporated into (47), yielding:

$$dt_M = dt_m \sqrt{1 - \frac{v^2}{c^2}}$$

$$dr_M \sqrt{1 - \frac{v^2}{c^2}} = dr_m$$

$$ds^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 dt_m^{\ 2} - \left(1 - \frac{v^2}{c^2}\right)^{-1} dr_m^{\ 2} - r_m^{\ 2} d\vartheta_m^{\ 2} - r_m^{\ 2}\left(\sin^2 \vartheta_m\right) d\phi_m^{\ 2} \tag{48}$$

Assume the m observers don't know about M. They might conclude that the passing M framework is moving for a number of possible reasons. For example, there might be a rocket attached to it, or it might be one mass in a system that is exchanging momentum with other masses. Equation (48) is a general equation describing the invariant interval of any framework which acquires a velocity in the r direction relative to m. In this case, the cause of the velocity v is a gravitational field and this velocity is a function of r.

AN APPROXIMATE VELOCITY EQUATION

Newton's Laws are used to determine the velocity equation for use in (48) by calculating the escape velocity of an object leaving a massive planet.

$$F = \frac{dP}{dt} \tag{49}$$

P = momentum of an object = mv
F = force on the object = $\frac{GMm}{r^2}$
G = gravitational constant
M = mass of planet
m = mass of escaping object
r = radius of m from center of M = $R + x$
v = velocity of object

The radius r will be divided into two segments: R is a specified radius (for example, the radius of the surface of M) and x is the object distance above R. The object m starts at R and is shot upwards (x direction away from M). The resulting equation for the position of the object is:

$$m\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = \frac{-GMm}{(x+R)^2} \tag{50}$$

Using the initial condition that the object velocity is v_o at position $x = 0$ defines the integration constant and gives the equation for object velocity as a function of position x.

$$v^2 = v_o^2 - \frac{2GM}{R} + \frac{2GM}{x+R} \qquad (51)$$

Noting that $v = 0$ at position $x = \infty$ gives the equation for escape velocity v_{oe}.

$$v_{oe} = \sqrt{\frac{2GM}{R}} \qquad (52)$$

The escape velocity is also the velocity that mass m would reach if it fell from $x = \infty$ to $x = 0$. If (51) is specified to be the velocity profile of mass m as it falls from $r = \infty$ (if v_o is chosen to be equal to v_{oe}), then (51) becomes:

$$v^2 = \frac{2GM}{r} \qquad (53)$$

If (53) is inserted into (48), the Schwarzschild Metric (46) is the result. The idea of a black hole comes from the condition $r = R_S$ and $R_S = \frac{2GM}{c^2}$. Under this condition, the invariant interval is no longer invariant. The Schwarzschild Radius R_S is a point of mathematical singularity and the behavior of the laws of physics becomes unclear at this location.

A BETTER VELOCITY EQUATION

Notice that the equations for velocity above are derived from Newtonian expressions for momentum and gravity (49). The Schwarzschild Metric (46) turns out to be a combination of relativistic and Newtonian components. A better metric would result by using a relativistic expressions for momentum and gravity.

$$P = \frac{m\,v}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad F = \frac{GM}{r^2}\left(\frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}\right) \qquad (54)$$

$$\frac{m}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = \frac{-GMm}{(x+R)^2\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v = c\sqrt{1-\exp\left(-\frac{2GM}{(x+R)c^2}\right)} \tag{55}$$

Noting $v = v_{oe}$ at position $x = 0$ gives the equation for escape velocity v_{oe}.

$$v_{oe} = c\sqrt{1-\exp\left(-\frac{2GM}{Rc^2}\right)} \tag{56}$$

The escape velocity from R_S is 0.795c. Including relativity into the velocity expression for a falling object eliminates the singularity which results in the concept of 'black hole.'

THE EXACT VELOCITY EQUATION

As has been shown in the article *Gravity and Energy*, an object falling within a spherical gravitational field gains energy exponentially. Using (34), the momentum P of this object changes as its kinetic energy KE changes and the kinetic energy change is equal to the potential energy change (W_{AB} per previous discussions) within the gravity field.

$$P = \left(m + \frac{KE}{c^2}\right)v = m\left(1+\frac{W_{AB}}{mc^2}\right)v = mc\beta\exp(\varepsilon) \tag{57}$$

$$\varepsilon = \frac{GM}{c^2}\left(\frac{1}{r}-\frac{1}{r_0}\right)$$

$r_0 =$ Initial radius from which mass m (frame A) starts falling
$r =$ Final radius of mass m as it falls

Note that $\beta = \frac{1}{c}\frac{dr}{dt}$ and $v = \frac{dr}{dt}$.

$$F = \frac{GM}{r^2}\left(m + \frac{KE}{c^2}\right) = \frac{GMm}{r^2}\left(1+\frac{W_{AB}}{mc^2}\right) = \frac{GMm}{r^2}\exp(\varepsilon)$$

$$mc\left(\frac{d(\beta\exp(\varepsilon))}{dt}\right) = -\frac{GMm}{r^2}\exp(\varepsilon)$$

$$mc\left(\frac{dx}{dt}\beta + \frac{d\beta}{dt}\right) = -\frac{GMm}{r^2}$$

Combining the above two equations gives:

$$m\frac{dr}{dt}\frac{dv}{dr}=\frac{GMm}{r^2}\left(-1+\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right)$$

$$-\frac{GM}{r^2c^2}dr=\frac{\beta}{1-\beta^2}d\beta$$

$$\beta=\sqrt{1-\frac{1}{\exp\left(\frac{2GM}{c^2}\left(\frac{1}{r}-\frac{1}{r_0}\right)\right)}} \qquad (58)$$

For the case $r_0=\infty$, β_{0e} is the escape velocity and

$$\beta_{0e}=\sqrt{1-\frac{1}{\exp\left(\frac{2GM}{Rc^2}\right)}} \qquad (59)$$

As before, R is the radius of the planet of mass M. For the case where $R=0$, $\beta_{0e}=1$. For the case $R=2GM/c^2$, $\beta_{0e}=0.795$. It is possible for light and matter to escape from the Schwarzschild Radius. An object of mass m would need kinetic energy $0.6485mc^2$ to reach the escape velocity. This would also be the kinetic energy that the object would gain when falling from far away to the Schwarzschild Radius.

INTERPRETING THE RESULT

The proper interpretation of (46) starts with the realization that it is an invariant interval, a familiar tool in Special Relativity. See Figure 13, in which reference frame B is passing reference frame A with velocity v.

As a first experiment, the planets shown in Figure 13 will be ignored. What's left is a simple experiment with two inertial frames. As seen by frame B, if the origins of both frames are passing at the instant shown, coordinate L on the frame A axis will be in a length contracted position compared to the same coordinate in frame B. A gap results between the coordinates. The closer v gets to c, the closer the frame A coordinate L gets to the frame A origin. When v equals c, all frame A coordinates are contracted to the origin. In addition, frame B observes that all frame A clocks have stopped.

In inertial experiments, an observer standing at the origin of frame B cannot directly