The article *The Acceleration Law* presents the mathematical tools needed to work with acceleration in the Special Theory of Relativity. To establish a convenient framework for acceleration experiments, those tools will be formed into a mathematical definition of an accelerating reference frame.

Fundamental Experiment

Figure 11 shows an object accelerating relative to inertial reference frames i and ii.

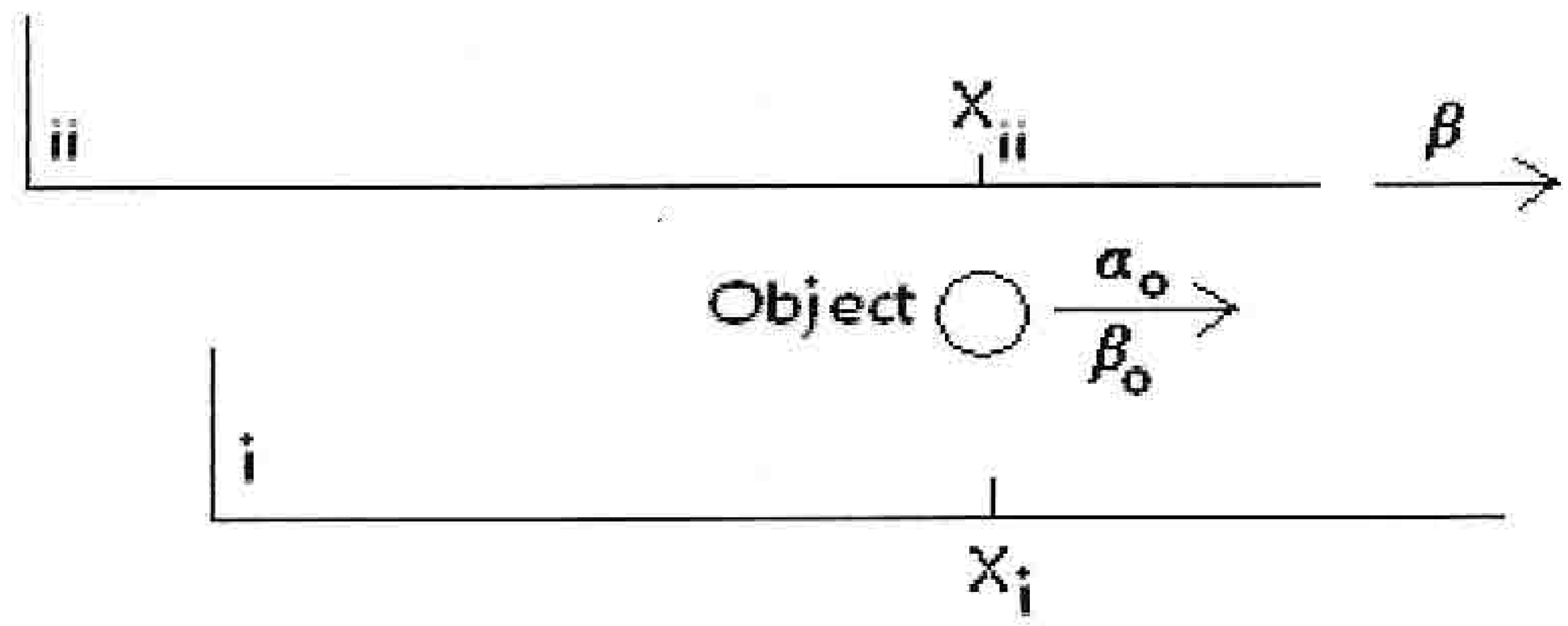

Figure 11. Single point object accelerates relative to frame i and frame ii.

Frame ii is moving with velocity β relative to frame i. Constant acceleration α_0 is 'felt' by observers on the object. The acceleration starts at coordinate zero in frame i with clocks reading zero on the object and frame i. The speed of light is $c = 3\times10^8\ m/s$.

Two main groups of information are desired in this experiment. The first is a mathematical description of how observers in inertial reference frames see the events of the acceleration of the object. This has already been shown in the article *The Acceleration Law*. The second information group is a mathematical description of how the accelerating observer sees the rest of the universe (which for this discussion will mean inertial reference frames in the universe).

One possible calculation technique to determine this second information group is to say that as the accelerating observer becomes stationary in inertial reference frame ii, he sees the rest of the universe in the same way as all the other observers in that inertial reference frame. This could be a plausible definition for an accelerating single observer, since this observer is only concerned with events which he can witness (happening next to him). But it is not true if the accelerating object has length (measured in the direction of the acceleration) with observers lined up along that length. In this situation, all the accelerating observers are concerned about events happening next to them all along the length of the object.

To define an accelerating reference frame, the experiment of Figure 11 is expanded to an experiment of a simple acceleration of an object with length. See Figure 12, where an observer standing at the origin of frame o is at the instant where acceleration α_0 begins. It will be specified that frame o is an accelerating reference frame attached to this observer. When the acceleration starts, all clocks read zero and the observer is stationary at the origin of an inertial reference frame i.

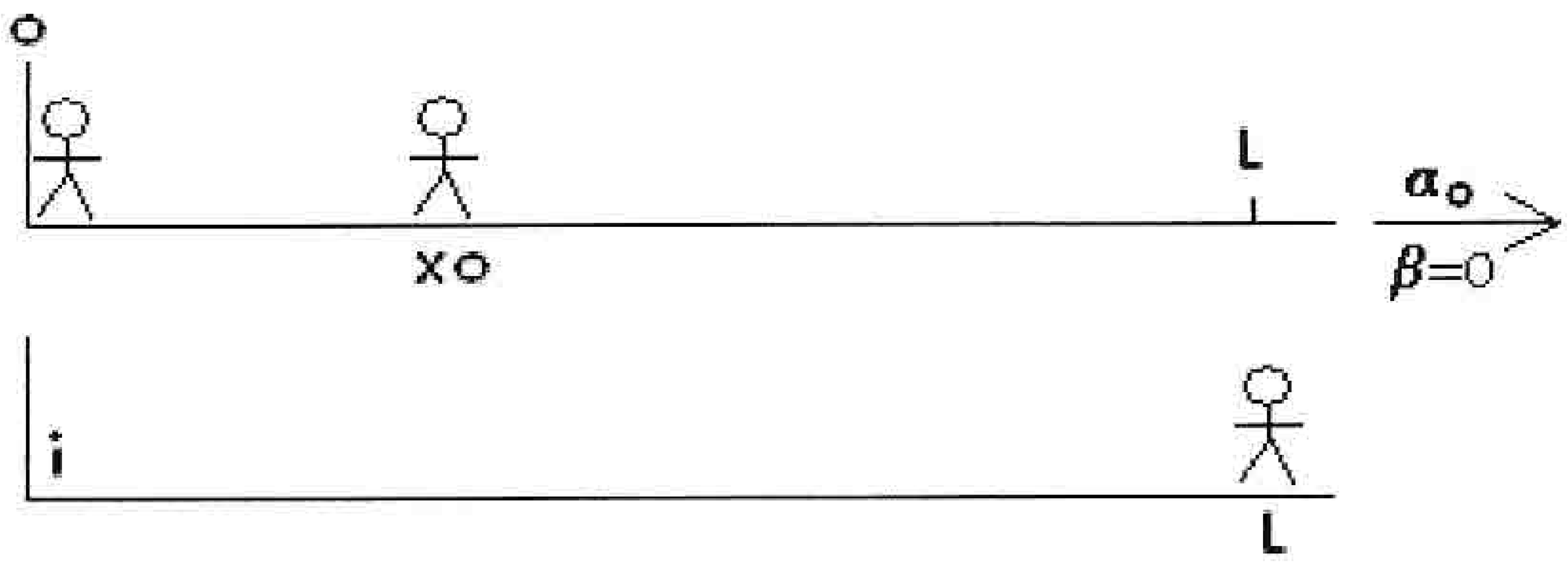

Figure 12. Starting point of acceleration of two different observers.

This experiment will be easier to understand if values are given to the events. The stationary reference frame i will measure a time period of 0.1 seconds for the origin observer to accelerate to $\beta = 0.95$. The value of $\alpha_0 = 30.42\ \sec^{-1}$ is found from (14b). After the accelerating observer has reached velocity β, his view of the experiment is shown in Figure 13.

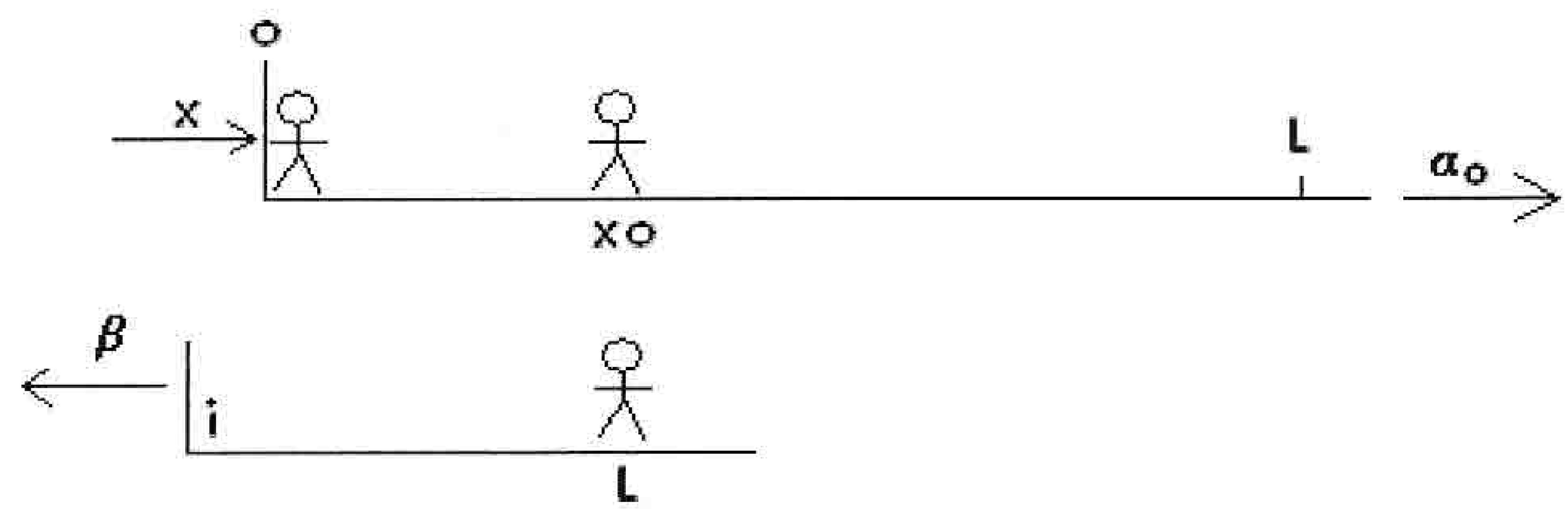

Figure 13. The accelerating observer view of the experiment after reaching speed β.

In Figure 13, frame i is passing the accelerating frame at a speed of $\beta = -0.95$. Length contraction has shortened frame i so that the frame i observer at coordinate L is now opposite coordinate xo of inertial reference frame ii in which the accelerating observer is momentarily stationary. Let $L = 1 \times 10^{10}$ *meter*. The accelerating observer has traveled a distance x as measured by frame i during this acceleration. This distance is given by (15) and:

$$x = \frac{c}{\alpha_o}\left[\sqrt{1+(\alpha_o t)^2} - 1\right] = 2.172 \times 10^7 \ meter$$

$$xo = (L - x)\sqrt{1-\beta^2} = 3.12 \times 10^9 \ meter \qquad (34)$$

In Figures 12 and 13, a second observer is standing at coordinate xo in frame i and starts accelerating independently at the same instant as the first observer. The acceleration value for the second observer at position xo will be specified by (21) from *The Acceleration Law* as:

$$\alpha_{xo} = \frac{30.42}{1 + \frac{30.42\left(3.12 \times 10^{9}\right)}{3 \times 10^{8}}} = 0.096 \text{ sec}^{-1} \tag{35}$$

From (14b), the acceleration time measured by frame i for the second observer to reach speed β is:

$$t' = \frac{\frac{.95}{.096}}{\sqrt{1 - .95^2}} = 31.69 \text{ sec} \tag{36}$$

The clock readings of both accelerating observers at the instant that they reach speed β are found from (16):

origin observer

$$t_o = \frac{1}{30.42} \ln\left[3.042 + \sqrt{1 + 3.042^2}\right] = 0.06 \text{ sec}$$

xo observer

$$t_{xo} = \frac{1}{0.096} \ln\left[3.042 + \sqrt{1 + 3.042^2}\right] = 19.08 \text{ sec} \tag{37}$$

The Accelerating Reference Frame

In Figure 13, consider the inertial reference frame ii superimposed upon frame o and passing frame i with velocity β. Both accelerating observers will simultaneously become stationary in this inertial reference frame. At that instant, they are a distance xo apart, just as they were at the start of the acceleration in frame i. However, their clock readings (37) are different at the instant they become stationary in this moving frame. If all the clocks in this inertial frame read zero at the start of the acceleration, these clocks would all read $0.1\sqrt{1 - 0.95^2} = 0.0312$ sec as the origin observer became stationary in this inertial reference frame. An observer at position xo in this inertial reference frame would compare his clock reading to that of the accelerating xo observer and get a difference of 19.05 sec. An observer next to the accelerating origin observer would only see 0.029 sec difference.

For an accelerating object with length, an inertial reference frame in which the origin observer is instantaneously stationary does not have the same clock readings along the object length as co-accelerating observers observing (21), even though the actual coordinates do match up between the object and inertial reference frame at the instant the object is stationary. If there were an infinite number of co-accelerating observers lined up from the origin to coordinate L in frame o at the start of the acceleration, these observers could be considered to be functionally equivalent to a reference frame accelerating with the origin observer. The events they see are symmetrical in the acceleration to any inertial reference frame and back in the reverse direction to the original stationary reference frame, as was discussed in *The Acceleration Law.* The events these observers see can be transformed to other reference frames.

This accelerating reference frame could be characterized as infinitely stiff, a condition which is normally not compatible with Special Relativity. If someone pushes on the end of an infinitely stiff object, the force is instantly felt at the other end of the object. This is contrary to the stipulation in Special Relativity that no information can travel faster than the speed of light. Note that in the above definition of an accelerating reference frame, no information travels faster than the speed of light, even though the reference frame appears to be infinitely stiff.

This definition of an accelerating reference frame is a shorthand notation for defining the theoretically ideal acceleration of objects with length. By specifying how an accelerating reference frame is constructed, any questions about the results obtained when using an accelerating reference frame can be answered. If acceleration of real objects with length is to be studied, the acceleration of this ideal (pseudo-infinitely stiff) object would be a simplified starting point that reveals the underlying principles. An object more nearly like real objects (with finite stiffness) could then be described as its behavior departs from this ideal case.