In the article *The Acceleration Law*, Special Relativity was used to show how coordinates on accelerating objects varied with the distance L along the object. Other characteristics such as velocity and acceleration were also shown to vary along the length of an accelerating object. Force and energy can be contained within accelerating objects. To see how they are influenced by acceleration, consider the rocket (accelerating reference frame) shown in Figure 14.

Fundamental Experiment

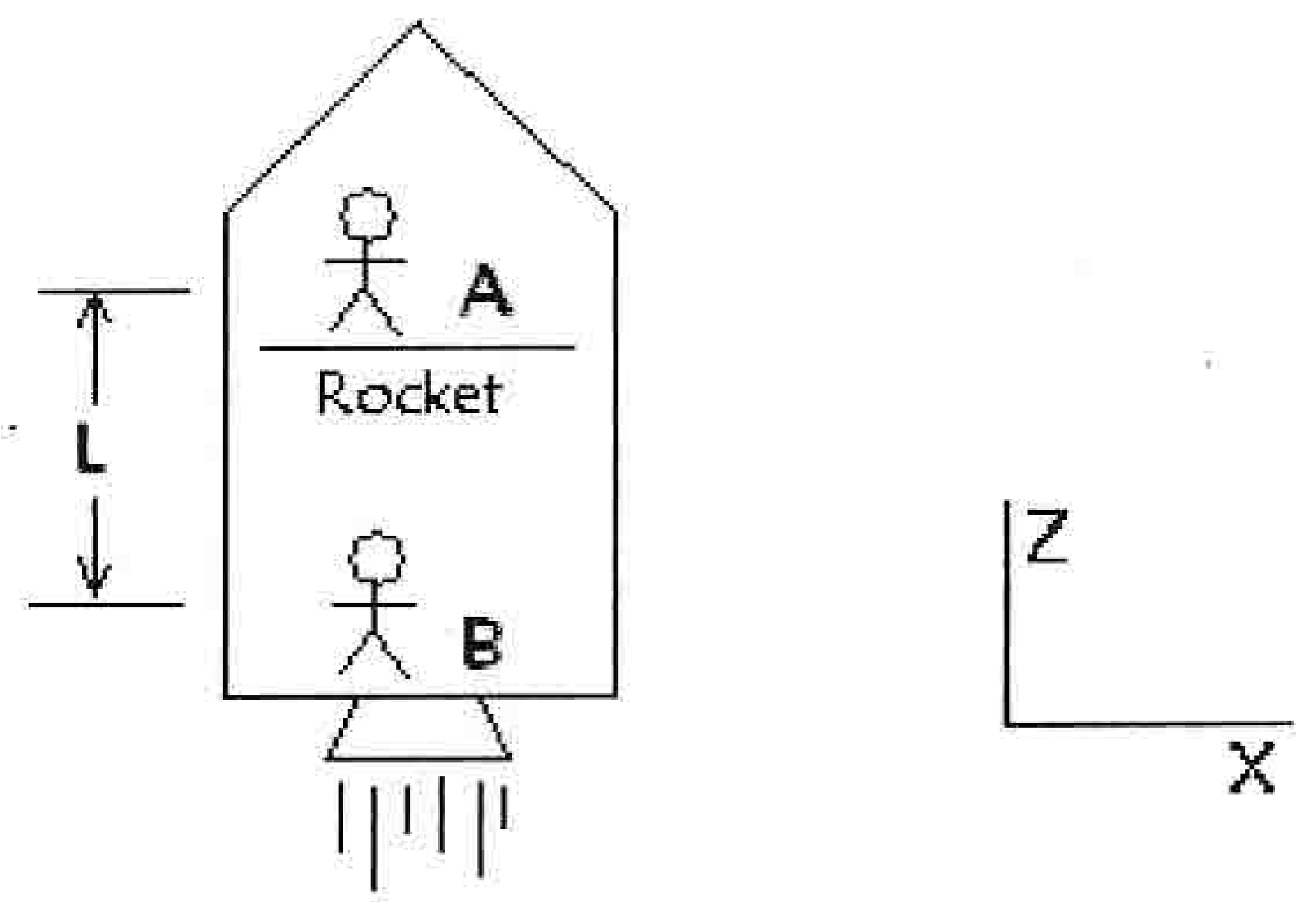

Figure 14. A rocket accelerates relative to an inertial reference frame.

Observer B will have acceleration α_B and observer A will have acceleration α_A as defined by (21).

Vertical Forces

Assume each observer holds a mass m during the rocket acceleration. If the accelerations at A and B are different, then the force applied to each mass is different. The change in momentum for each mass as it goes from a velocity of zero to velocity β is the same. So, the force applied to each mass multiplied by the time interval of the force application (31) would be the same for both masses.

$F_A = m\alpha_A c$ for mass at A $\qquad$ $F_B = m\alpha_B c$ for mass at B

$$F_A t_{OA} = F_B t_{OB}$$

$$F_A = \frac{F_B}{1 + \frac{\alpha_B L}{c}} \tag{38}$$

During the constant acceleration, suppose observer B has a massless rod that he uses to reach 'up' to the mass at A. This mass is accelerating with observer A, but now is supported by observer B. The force 'felt' by this mass is F_A. The force 'felt' by observer B on his end of the rod is F_{AB}. When the acceleration is over, this mass is going the same velocity as observer B and the distance

L is the same. The impulse applied to the end of the rod by observer B and the impulse received by mass m must be equal. Using (31):

$$F_A t_{OA} = F_{AB} t_{OB}$$

$$F_B = F_{AB} \tag{39}$$

Notice that the force observer B feels on the end of his rod, F_B, is the same for any value of L. Consequently, if observer B began with $L = 0$ and slowly 'raised' mass m up to observer A with his rod, the force that observer B feels would be constant throughout the process. The work expended by observer B for this process is W_{AB}.

$$W_{AB} = F_B L \tag{40}$$

Consequently, the kinetic energy that the mass would generate if it 'fell' from A to B can also be calculated. From *The Acceleration law*, it is already known that observer A and observer B are instantaneously stationary in any inertial reference frame simultaneously a distance L apart. So, if mass m falls from A to B, it becomes stationary in that inertial frame and observer B accelerates to it. The velocity of observer B after he travels a distance L is β_L and is calculated using (14) and (15).

$$\beta_L = \frac{\alpha_B t}{\sqrt{1+(\alpha_B t)^2}}$$

$$(\alpha_B t)^2 = \left(1+\frac{\alpha_B L}{c}\right)^2 - 1$$

$$\sqrt{1-\beta_L^2} = \frac{1}{1+\frac{\alpha_B L}{c}} = \frac{\alpha_A}{\alpha_B} \tag{41}$$

The kinetic energy KE received by observer B as mass m impacts him is:

$$KE = mc^2\left(\frac{1}{\sqrt{1-\beta_L^2}} - 1\right) = mc^2\left(\frac{\alpha_B L}{c}\right) = F_B L \tag{42}$$

The kinetic energy received from the fall is equal to the work required to lift mass m during the acceleration. The force felt in the direction of acceleration of a mass supported at B but accelerating at A is given by (38). This holds for any L and any constant acceleration.

Horizontal Forces

A comparison of length dimensions in the "horizontal" direction within the rocket is shown in

Figure 15.

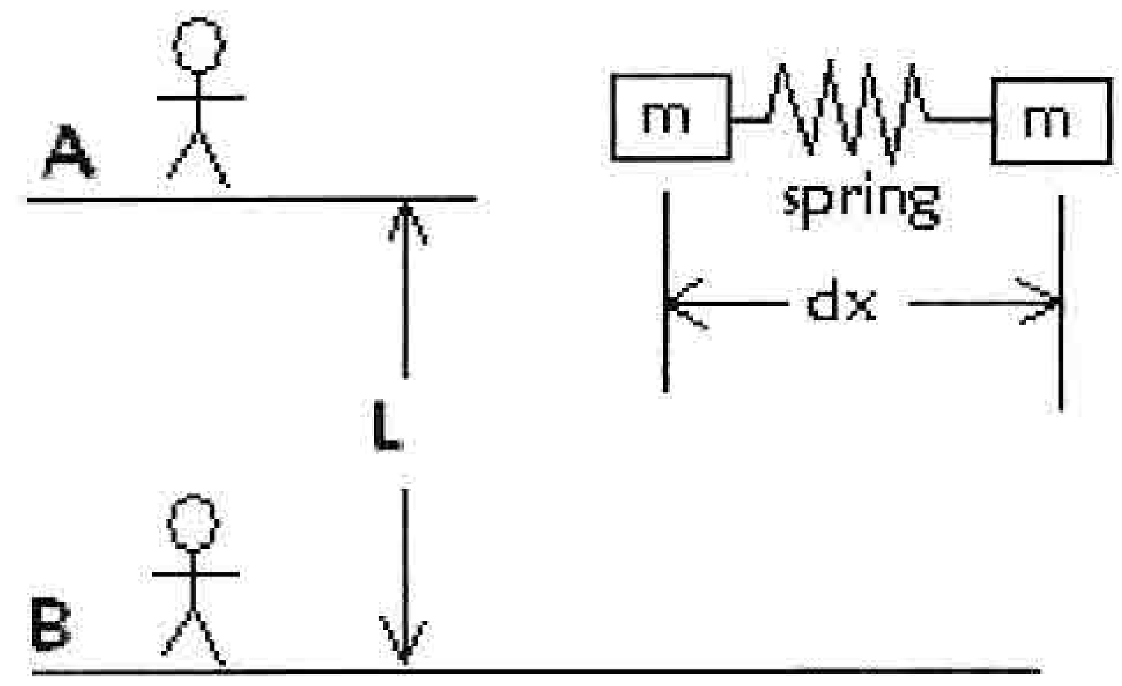

Figure 15. Horizontal dimensions in the two locations within the rocket.

During the acceleration, if observer A "drops" a mass down to observer B, it would be expected that the mass would travel straight down in the z direction, there being no reason it would move left or right. Similarly, two masses dropped from A to B would both be expected to fall straight (neglecting their small gravitational attraction for each other). But, how can the observers be sure that the distance between the masses is measured to be the same in A and B?

The way this is accomplished is to place a spring between the two masses. As the two masses move from A to B, the amount of energy recovered from the spring would have to be zero. The only energy received by observer B is the potential energy difference generated by the masses as they travel between levels A and B. Any additional spring energy recovered would violate the Law of Conservation of Energy. This result can be summarized in the transformation:

$$dx_A = dx_B \tag{43}$$

To determine the relationship of forces applied perpendicular to the acceleration direction, see Figure16, where an observer at level B pushes horizontally on a board.

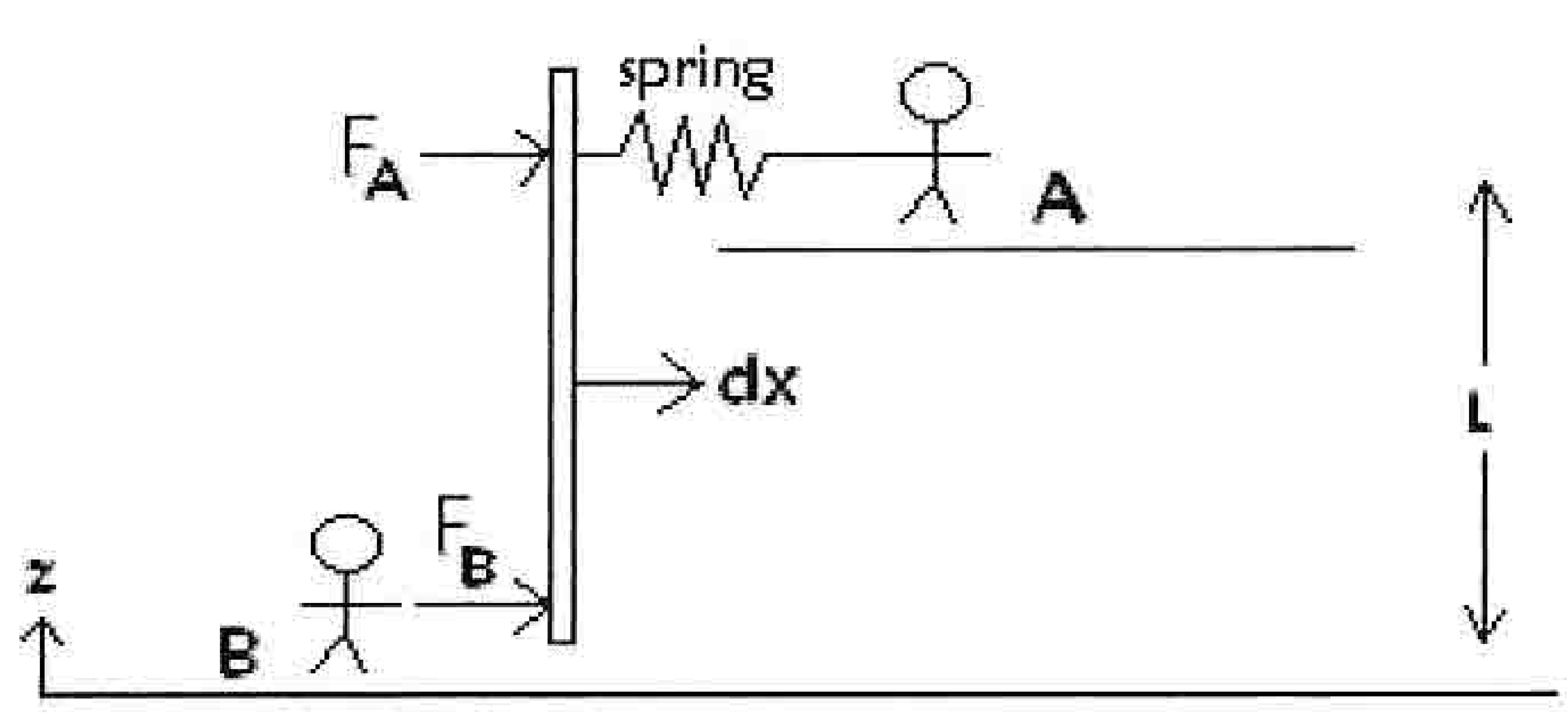

Figure 16. Experiment of horizontal force transmission between rocket levels.

Observer B pushes with force F_B and observer A feels force F_A. The spring that observer A is holding is compressed. The spring force-displacement relationship is $F = kdx$, where k is the spring constant and dx is the displacement of the board against the spring. The board compresses the spring, resulting in spring potential energy $E = \frac{1}{2} F_A dx$. The energy expended by the observer is $\frac{1}{2} F_B dx$. The spring is clamped in its compressed position and relocated to frame B. The energy E in this compressed spring has mass $m = E / c^2$. This mass has an acceleration "weight" measured to be F_{WA} in frame A. The "weight" felt by observer B as he lowers the spring to frame B is a constant $F_{WB} = m\alpha_B c$ by the same reasoning as explained for (39) and (40).

Continuing with that reasoning, the work expended by observer B as he lowers the compressed spring to frame B is W_{AB}.

$$W_{AB} = F_{WB} L = m\alpha_B cL \qquad (44)$$

So, the energy that observer B finds in the compressed spring would be the energy he expended to compress the spring minus the energy he recovered as he lowered the spring to frame B.

$$\frac{1}{2} F_A dx = \frac{1}{2} F_B dx - \frac{E}{c^2} \alpha_B cL = \frac{1}{2} F_B dx - \frac{\frac{1}{2} F_A dx}{c^2} \alpha_B cL \qquad (45)$$

$$F_A = \frac{F_B}{1 + \frac{\alpha_B L}{c}} \qquad (46)$$

This is the horizontal force transformation between rocket levels A and B. It is identical to the vertical force transformation (38).

The Acceleration Law and Energy

Energy transfer between levels A and B can show other interesting information about the Special Relativity Acceleration Law. With the spring energy relocated to level B, (44) and the Law of Conservation of Energy (45) can be combined.

$$W_{AB} = \left(\frac{E}{c^2} \right) \alpha_B cL$$

$$\frac{1}{2} F_B dx = \frac{1}{2} F_A dx + W_{AB} = E + W_{AB}$$

$$\frac{F_B}{F_A} = 1 + \frac{W_{AB}}{E}$$

$$\alpha_A = \frac{\alpha_B}{1 + \dfrac{W_{AB}}{E}} \qquad (47a)$$

$$\alpha_A = \frac{\alpha_B}{1 + \dfrac{W_{AB}}{mc^2}} \qquad (47b)$$

The Special Relativity Law of Acceleration is also a function of energy. The geometry of the acceleration experiment is influenced by the Law of Conservation of Energy for the same reason that it influenced in Newtonian dynamics: geometry and force combine to give energy.

Summary

Forces between different coordinates on an accelerating object are influenced by their relative positions on the object, but not their direction. This requires care in calculating the energy that those forces produce as they move about the accelerating object. The effect of acceleration on this energy calculation turns out to be a fundamental relationship in the Special Relativity Law of Acceleration.